HOW TO PROVE THE POWER RULE FOR DERIVATIVE FUNCTIONS USING FIRST PRINCIPLES

BY

M.A. TOLENTINO

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POWER RULE

'binomial theorem'

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$= \binom{n}{0} a^{n-0} b^0 + \binom{n}{1} a^{n-1} b^1 + \ldots + \binom{n}{n} a^{n-n} b^n$$

$$\left\{ \binom{n}{k} = \frac{n!}{k!\,(n-k)!} \right\}$$

$$A(x) = x^n$$

$$A'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\left(\sum_{k=0}^{n} \binom{n}{k} x^{n-k} h^k \right) - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\left(\binom{n}{0} x^{n-0} + \binom{n}{1} x^{n-1} h^1 + \binom{n}{2} x^{n-2} h^2 + \ldots + \binom{n}{n} x^{n-n} h^n \right) - x^n}{h}$$

$$= \lim_{h \to 0} \frac{1\cancel{x^n} + n x^{n-1} \cancel{h} + \binom{n}{2} x^{n-2} \cancel{h}^2 + \ldots + \cancel{h}^n - \cancel{x^n}}{\cancel{h}}$$

$$= \cancel{\lim_{h \to 0}} \; n x^{n-1} + \binom{n}{2} x^{n-2} \underset{\times 0}{\underset{\downarrow}{h}} + \binom{n}{3} x^{n-3} \underset{\times 0}{\underset{\downarrow}{h^2}} + \ldots + \underset{0}{\underset{\downarrow}{h^{n-1}}}$$

$$A'(x) = n x^{n-1}$$